CRANFIELD UNIVERSITY

Mathieu Farges

DYNAMICS OF FLUIDIC ENERGY DEVICES (DFED)

ASSIGNMENT

MSc Renewable Energy Engineering

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Module Leader: Dr Takafumi Nishino

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Nomenclature

a	axial induction factor
B	Number of blades of rotor
B_A	Array blockage ratio
B_G	Global blockage ratio
B_L	Local blockage ratio
c	Chord length (m)
C_D	Drag coefficient
C_L	Lift coefficient
C_P	Power coefficient
C_T	Thrust coefficient
p	Local pressure (Pa)
r	Distance from the hub (m)
S	Rotor's swept area (m^2)
U	Upstream wind speed (m/s)
u_2	Wind speed near the rotor (m/s)

Greek symbols

ρ	Density (kg/m^3)
φ	Angle of relative wind (rad)

1. INTRODUCTION

Since the amount of energy extractable from a flow is proportional to the density of the fluid, tidal energy can be considered as a promising way to get clean and sustainable energy. Indeed, sea water is about thousand times denser than air and tidal current is more regular than wind current, which lead to consider tidal energy at least as credible as wind energy.

Tidal turbines cannot be simply considered as “under water wind turbines”. They are often installed in shallow water, in a limited lagoon, and many turbines are placed close together. Thus, the tidal turbines cannot be considered as placed in a wide space: interaction between turbines and spatial boundaries are not negligible. Garrett and Cummins (2007) [1] have shown that a “blockage effect” increases the extractible power when the rotor swept area becomes close to the cross-sectional area of flow passage.

The objectives of this report are to explain the influence of the blockage effect on the performance of a single turbine in a water channel, and then in an array of several turbines in a wider water channel.

2. ANSWERS TO THE QUESTIONS

A. Single turbine in a water channel

A.1

In order to estimate the flow loads in a turbine, we need to define several coefficients:

- C_T is the thrust coefficient, defined as :

$$C_T = \frac{Thrust}{\frac{1}{2}\rho U^2 S} \quad [eq. 1]$$

where, *Thrust* is the force perpendicular to the rotor plane, U is the upstream flow speed, ρ is the mass density of the flow, and S is the rotor's swept area.

- C_P is the power coefficient, defined as :

$$C_P = \frac{Power}{\frac{1}{2}\rho U^3 S} \quad [eq. 2]$$

where, *Power* is the power extracted by the rotor due to the torque force (W).

In order to increase the energy yield of a turbine, one have to increase its power coefficient C_P.

- *a* is the induction factor, defined as :

$$a = \frac{U - u_2}{U} \quad [eq. 3]$$

where, U is the upstream wind speed, and u_2 the wind speed near the rotor, as shown in the figure 1 below:

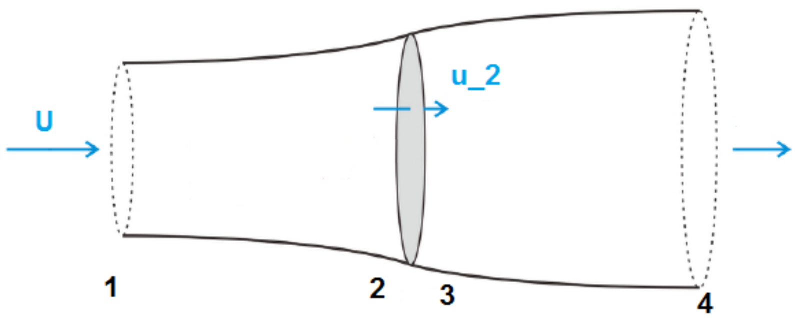

Figure 1 – Flow velocity in a turbine streamtube

The induction factor *a* can be seen also as a "perturbation factor", since it indicates how the velocity decreases through the turbine:

$$u_2 = U * (1 - a) \quad [eq. 4]$$

We will show in the section below that the value of C_P cannot be more than a certain limit.

Given the streamtube of figure 1, because the Bernoulli function is constant between 1 and 2, and between 3 and 4, we can write:

$$p_1 + \frac{\rho U^2}{2} = p_2 + \frac{\rho U^2 (1-a)^2}{2} \quad [eq. 5]$$

$$p_3 + \frac{\rho U^2 (1-a)^2}{2} = p_4 + \frac{\rho {u_4}^2}{2} \quad [eq. 6]$$

And $u_4 = U(1-2a)$ $[eq. 7]$ [reference 2]

Then $p_2 - p_3 = 2\rho U^2 a(1-a)$ $[eq. 8]$

And the power extracted is:

$$Power = \left(\frac{\rho U^3 S}{2}\right) * 4a(1-a)^2 \quad [eq. 9]$$

Thus

$$C_P = 4a(1-a)^2 \quad [eq. 10]$$

The maximum value of C_P is achieved at a = 1/3, and is equal to 16/27.

Considering this so called “actuator disk theory”, when the channel’s cross-sectional area is much larger than the rotor’s swept area,

$$C_{Pmax} = 16/27 \quad [eq. 11]$$

This maximum value of 16/27 for C_P is called the “Betz limit”.

A.2

Here, we do not consider that the channel's cross-sectional area is much larger than the rotor's swept area anymore.

The local blockage ratio B_L defines how close the channel's cross-sectional area and the rotor's swept area are:

$$B_L = \frac{turbine's\ frontal\ projected\ area}{channel\ cross\ sectional\ area} \quad [eq. 12]$$

Garrett and Cummins (2007) have demonstrated how this local blockage ratio B_L affects the maximum power coefficient of an ideal actuator disk [1]:

$$C_{Pmax} = \frac{16}{27} * (1 - B_L)^{-2} \quad [eq. 13]$$

When the channel's cross-sectional area is close to the rotor's swept area, not only the kinetic energy is extracted: but also potential energy from the flow. That is why **the maximum power coefficient C_{Pmax} of a turbine increases when B_L increases**, as one can easily see with equation 13.

Thus, the local blockage ratio B_L influences the relation between the induction factor *a*, and the coefficients C_T and C_P:

- When B_L becomes higher, the value of C_T increases for a given value of a ;
- When B_L becomes higher, the value of C_P increases for a given value of a.
- As the value of B_L increases, the maximum value of C_P is achieved at higher values of a (as shown in the table below).

Table 1 - Influence of B_L in the relation between C_P and a

B_L	C_{Pmax}	Value of *a* for C_{Pmax}
0.1	0.73	0.40
0.2	0.93	0.45
0.3	1.21	0.50
0.4	1.64	0.55

Figures 2 and 3 in the appendix represent “C_T versus a” and “C_P versus a” graphs for different values of B_L, in which one can appreciate the influence of B_L in those relations.

A.3

The results achieved above can lead to a discussion about the optimal design of a turbine rotor when it operates at high blockage conditions (B_L =0.4 for example). The aim here is to achieve a C_P value as high as possible for a given B_L.

As B_L is fixed, we cannot talk about changing the length of the blades in our discussion. Therefore, only very specific criteria derived from the high blockage condition are considered.

We can see in figure 3 that C_P reaches a maximum value for a specific value of the induction factor a. The value of the induction factor which corresponds to the maximum C_P increases when B_L increases. This leads to consider the induction factor *a* as an optimization criteria.

For each value of B_L, we can know the optimum value of a. The problem is: how to achieve this value of a in the rotor design? The axial induction factor can be changed by varying the chord length of the blade and the number of blades [4].

Let us consider some extreme cases. For zero blades or a zero value of chord length, one can easily see that there is no perturbation of the flow, thus a = 0. For an infinite number of blade or a very high chord length, the flow cannot pass through the turbine, and there is no velocity after this disk barrier, thus a = 1. From those extreme cases, one can assume that, in order to increase a, we can increase the number of blade and/or the chord length.

However, in order to reach the optimum value of a, an analytical expression of a is required. This kind of relation is not commonly given in the literature, and it is not easy to find one. Here is one from A. Sharifi and M.R.H. Nobari [5] :

$$a = \frac{1}{\frac{4\sin^2\varphi}{\frac{cB}{2\pi r}(C_l \cos\varphi + C_d \sin\varphi)} + 1} \qquad \text{[eq. 14]}$$

where, c is the chord length, B the number of blades, r the distance from the hub, φ the angle of relative wind, Cl and Cd the lift and drag coefficients.

From this equation, we can see that, when the number of blades or the chord length increases, the value of *a* increases. From Table 1, we know that the optimum value of *a* (induction factor) slightly increases when B_L becomes higher.

This leads to the conclusion that in order to achieve high C_P value, **we should increase the number of blades and/or the chord length of the turbine**.

As a first approximation, from the equation 14, it seems that reducing the angle of relative wind φ will roughly also lead to increasing a. Since the angle of relative wind φ is the sum of the section pitch angle and the angle of attack, a lower φ can mean (for the design) a **less twisted blade**, which can ease the manufacturing process.

However, all these conclusions have to be regarded with caution, as lift and drag coefficients can be linked themselves to the induction factor, and because a greater number of blades or chord length can lead to undesirable interference between the blades themselves.

A better way to optimize C_P may be to use a BEM (Blade element momentum) theory algorithm in order to evaluate the value of the induction factor a. For a given airfoil, we can calculate *a* for different values of chord length and/or number of blades. By plotting a 3D surface, with "chord length versus number of blades" in the horizontal plane, and the induction factor value in the vertical axis, one can find the most appropriate couple of (chord length , number of blade) in order to reach the desired value of a.

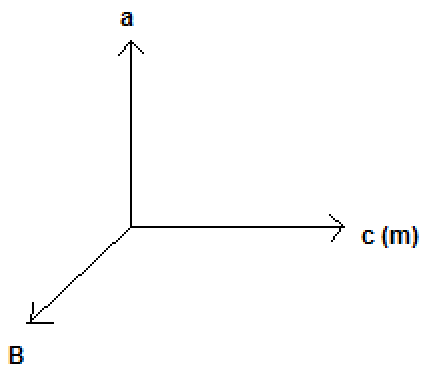

Figure 4 – Number of blade versus chord length versus a

A software can run this procedure for each value of r, using a mean value of chord length, in order to choose the number of blades. Once the number of blade is fixed, the procedure computes the appropriate chord length for each value of r.

Since we are looking for a value of a greater than 0.4, we also have to consider the Glauert correction in the BEM calculations.

B. Multiple turbines in a wide water channel

B.1

When we consider not only an isolated turbine but a row of several horizontal-axis turbines placed in a wide water channel, the local blockage ratio BL alone is not adequate to describe the blockage effect. We need three ratios:

- The local blockage ratio B_L is defined in the same way as before :

$$B_L = \frac{\pi d^2/4}{(d+s)*h} \quad [eq. 15]$$

- The array blockage ratio B_A represents the length ratio of the array in the channel :

$$B_A = \frac{n*(d+s)}{w} \quad [eq. 16]$$

- The global blockage ratio B_G defines the surface ratio of the turbine array in the channel :

$$B_G = \frac{n*\pi d^2/4}{h*w} \quad [eq. 17]$$

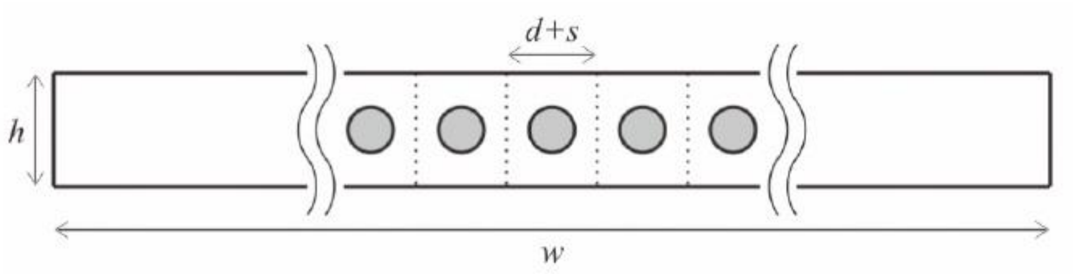

Figure 5 – Multiple turbine array

We also need to introduce two scale of flows:

- the device-scale flow corresponds to the streamtube passing through one turbine of the array

- the array-scale flow corresponds to the streamtube passing through the whole array of turbines as if it was a single large device

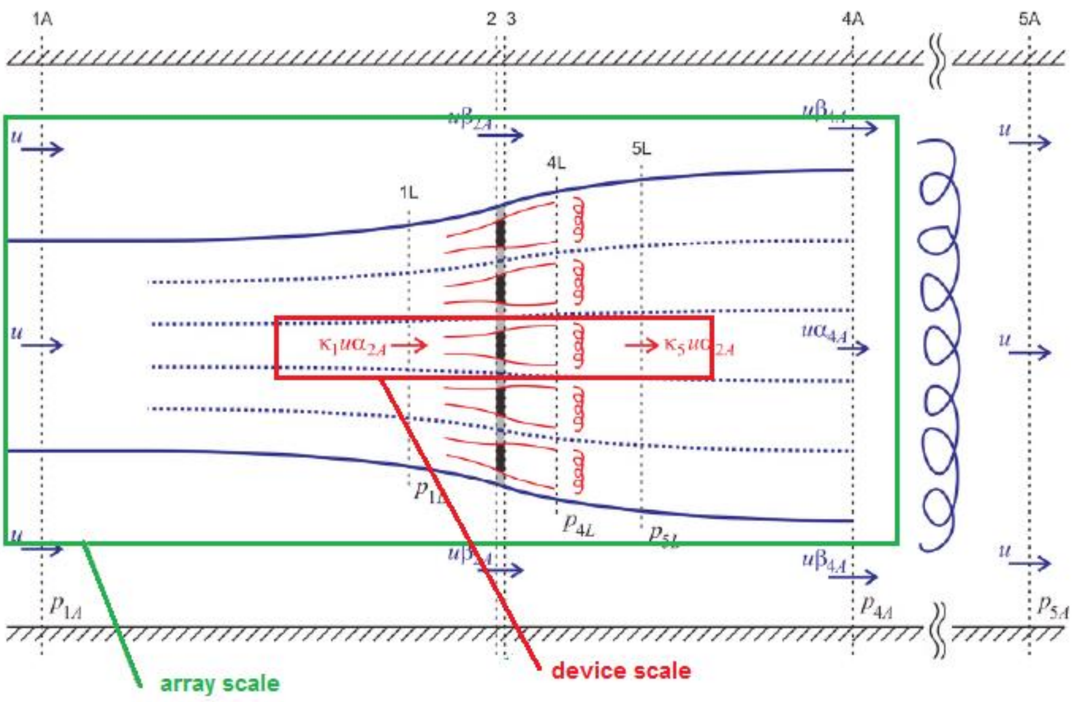

Figure 6 – Array scale and device scale flows (adapted from “Two-scale dynamics of flow past a partial cross-stream array of tidal turbines”, T. Nishino, R.H.J. Willden (2013), *J. Fluid Mech.*)

The scale separation assumption considers that the Garrett-Cummins (2007) model can be applied to the device-scale flow and the array-scale flow.

This assumption is satisfied when the number of turbine is large enough. More than 10 turbines can be considered as “large enough”.

B.2

Under the scale separation assumption described above, one can compute the upper limit of the power generated by a single row of a number of horizontal-axis turbines placed in the middle of a wide water channel.

By plotting C_{Pmax} versus B_L, one can find the maximum value of C_{Pmax} when the global blockage ratio B_G =0. The results are given in Figure 7.

In those figures, B_L vary from B_G to π/4, as B_L cannot be smaller than B_G by definition, and as the value of B_L here cannot be more than "*the area of a circle inscribed in a square under the area of this square",* which is equal to π/4.

When B_L equal B_G, it corresponds to a full fence array.

Figures 8 and 9 show the same type of plot than Figure 7, but with different values of B_L. A summary of the influence of B_G is given in Table 2.

The Matlab code given in the appendix computes, for a given value of BG, the "C_{Pmax} versus B_L" plot. The values of B_L considered in the plot are from B_G to Pi/4 (only values with physical meaning). The plot indicates the upper limit of C_{Pmax} and the specific value of B_L to achieve it.

Table 2 – Influence of B_G in the relation between C_{Pmax} and B_L

B_G	Upper limit of C_{Pmax}	Value of B_L to achieve it
0.0	0.798	0.41
0.1	0.954	0.47
0.2	1.168	0.54

3. CONCLUSIONS

Tidal energy turbines can be considered as a really promising way to get clean and sustainable energy, as its power coefficient can be greater than the Betz Limit (C_{Pmax} = 0.593), up to the Nishino-Willden limit (C_{Pmax} = 0.798) for a wide channel (B_G = 0.0), and even more when turbines get closer to full fence array (B_G increase).

For a single turbine, the power coefficient increase when the blockage ratio increase. By slightly increasing the number of blades and/or the chord length, one can optimize the power extracted by the turbine from the flow. For an array of turbines, the upper limit of C_{Pmax} increase when B_G increase, and can be reached for a specific value of B_L.

REFERENCES

[1] GARRETT, C. & CUMMINS, P. (2007) "The efficiency of a turbine in a tidal channel." *J. Fluid Mech.*, 588, pp. 243–251.

[2] Andrze J. WORTMAN (1983), *Introduction to wind turbine engineering*, Boston: Butterworth Publishers

[3] T. NISHINO, R.H.J. WILLDEN (2012), "The efficiency of an array of tidal turbines partially blocking a wide channel", J. Fluid Mech., 708, pp. 596–606.

[4] M. COLLU, "Wind Energy", Cranfield University lecture, 30 oct. 2014

[5] A. SHARIFI, M.R.H. NOBARI (2013), "Prediction of optimum section pitch angle distribution along wind turbine blades", *Energy Conversion and Management*, Elsevier, 67, pp. 342–350

APPENDIX

The Matlab code is given below:

```
% two-scale blockage effect %

%initialisation
B_g=0.00001; %B_g can't be zero
gm_l=linspace(0,1,1000);
gm_a=linspace(0,1,1000);
M=[]; %matrix of Cpmax
X=[]; %matrix of B_l

%first loop

kmax=79*(1-B_g); % B_L can't be higher than Pi/4=0.786

for k=1:kmax

B_l=B_g+k*0.01 % The local blockage B_l must not be smaller than the
global blockage B_l
B_a=B_g/B_l; % B_g=B_l*B_a

X=[X B_l]; % store the value of B_l for each iteration


for i=1:1000
a_l(i)=1-((1+gm_l(i))/((1+B_l)+sqrt((1-B_l)^2+B_l*(1-
1/gm_l(i))^2)));
Ct_l(i)=(1-gm_l(i))*(((1+gm_l(i))-(2*B_l)*(1-a_l(i)))/(1-(B_l*(1-
a_l(i))/gm_l(i)))^2);

for j=1:1000
a_a(j)=1-((1+gm_a(j))/((1+B_a)+sqrt((1-B_a)^2+B_a*(1-
1/gm_a(j))^2)));
Ct_a(j)=(1-gm_a(j))*(((1+gm_a(j))-(2*B_a)*(1-a_a(j)))/(1-(B_a*(1-
a_a(j))/gm_a(j)))^2);
Ct_a2(i,j)=((1-a_a(j))^2)*B_l*Ct_l(i);
e(i,j)=abs(Ct_a2(i,j)-Ct_a(j));
end

[min_err,p]=min(e(i,:)); % to find the a_a value that gives
"Ct_a2=Ct_a"
Cp_l(i)=(1-a_l(i))*Ct_l(i);
Cp_g(i)=((1-a_a(p))^3)*Cp_l(i);
a_g(i)=1-(1-a_l(i))*(1-a_a(p));

end

Cpmax=max(Cp_g) ;

M=[ M Cpmax ]; % store the value of Cpmax for each iteration

end
```

```
[ Cplim ind ] = max(M) ; % max Cp_max (and the index location of it
in the matrix) for the given B_g

Cplim % displays the max Cp_max

BL_opt=X(ind) % the specific value of BL to achieve the upper limit
of Cp_max

figure
plot(X,M)
xlabel('B_L')
ylabel('Cp_m_a_x')
hold on;
 plot(BL_opt,Cplim,'ks','markerfacecolor',[0 0 0]);
 text(BL_opt,Cplim-0.02,['Max: ', num2str( Cplim ), ' for BL: ',
num2str( BL_opt ) ] ) ; % indicates the optimum in the plot
```

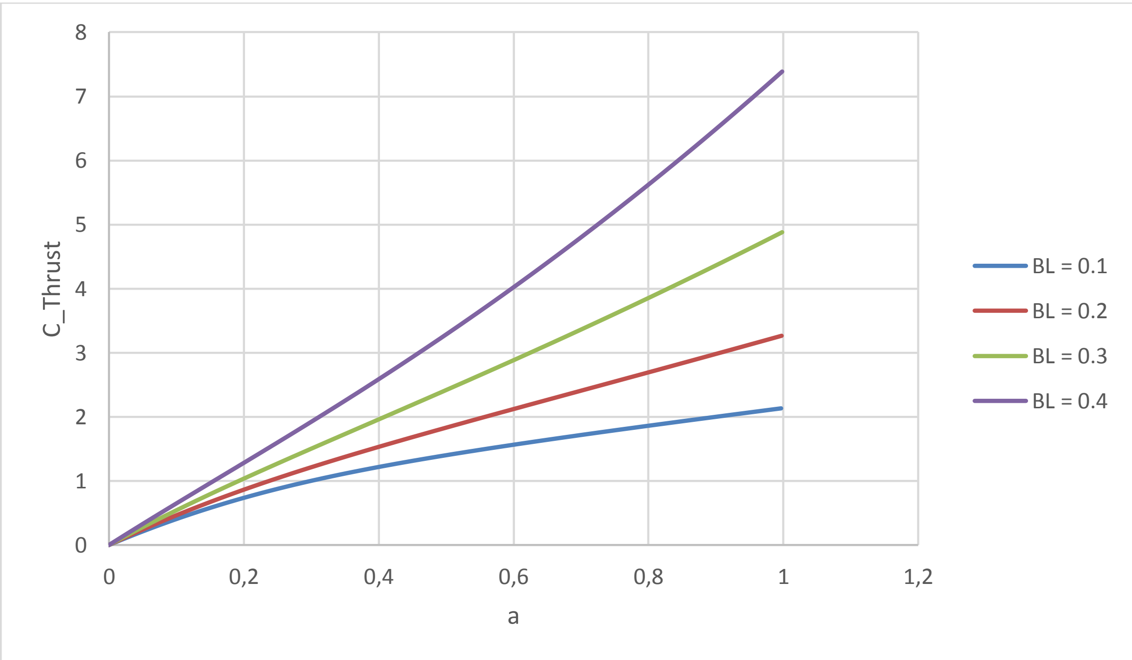

Figure 2 – C_T versus a for different values of B_L

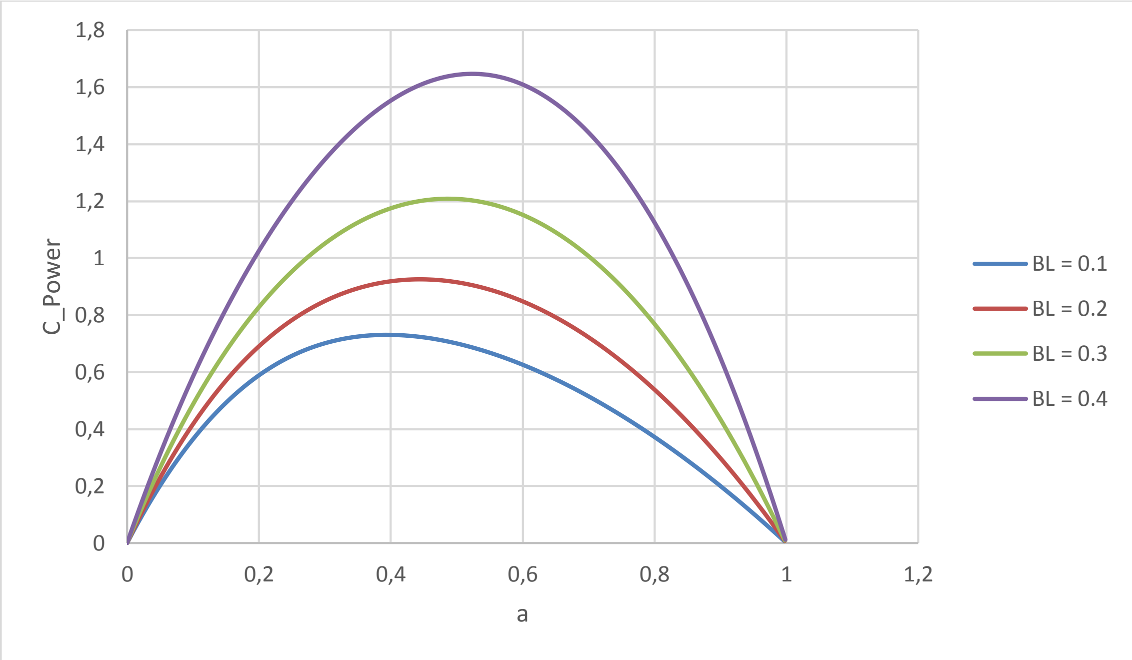

Figure 3 – C_P versus a for different values of B_L

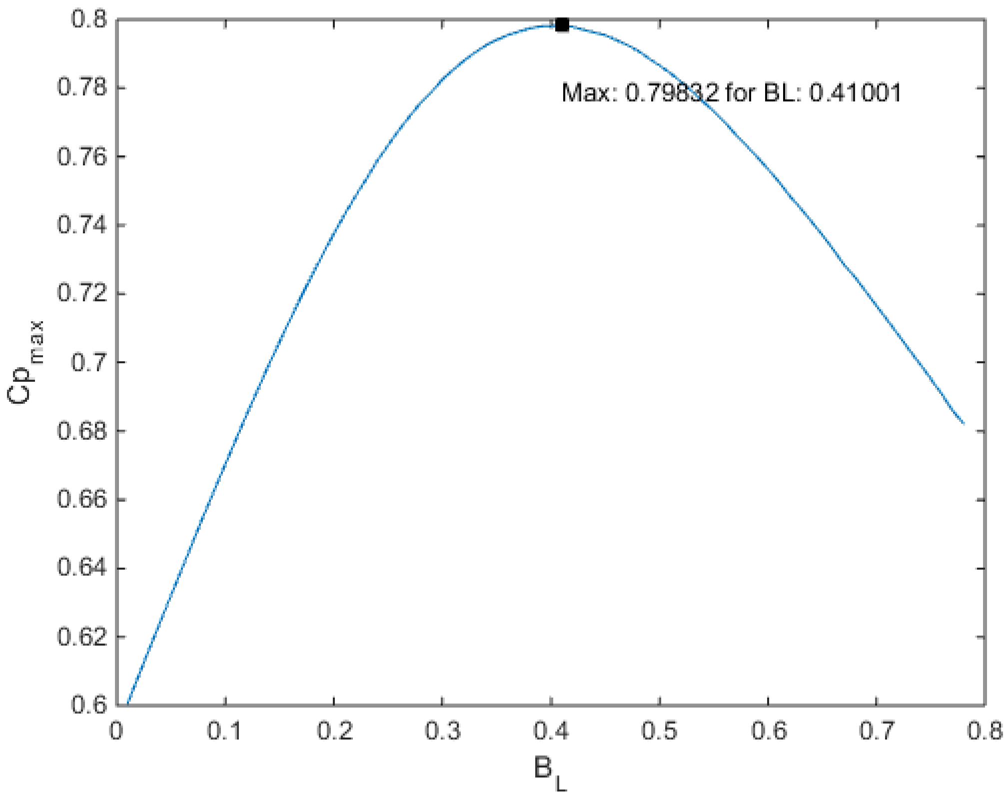

Figure 7 – C_{Pmax} versus B_L for B_G = 0.0

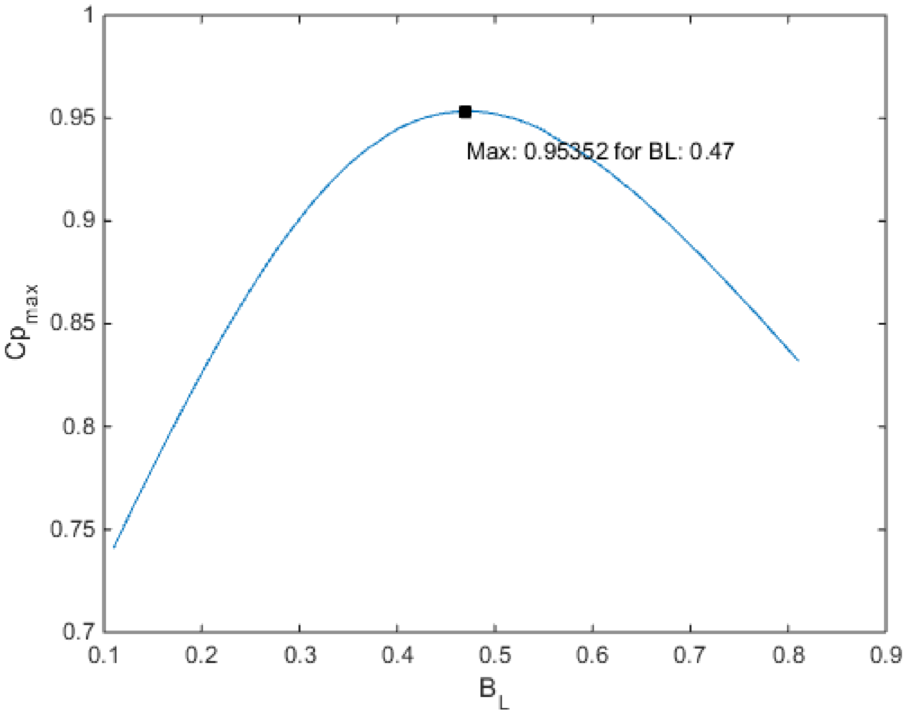

Figure 8 – C_{Pmax} versus B_L for B_G = 0.1

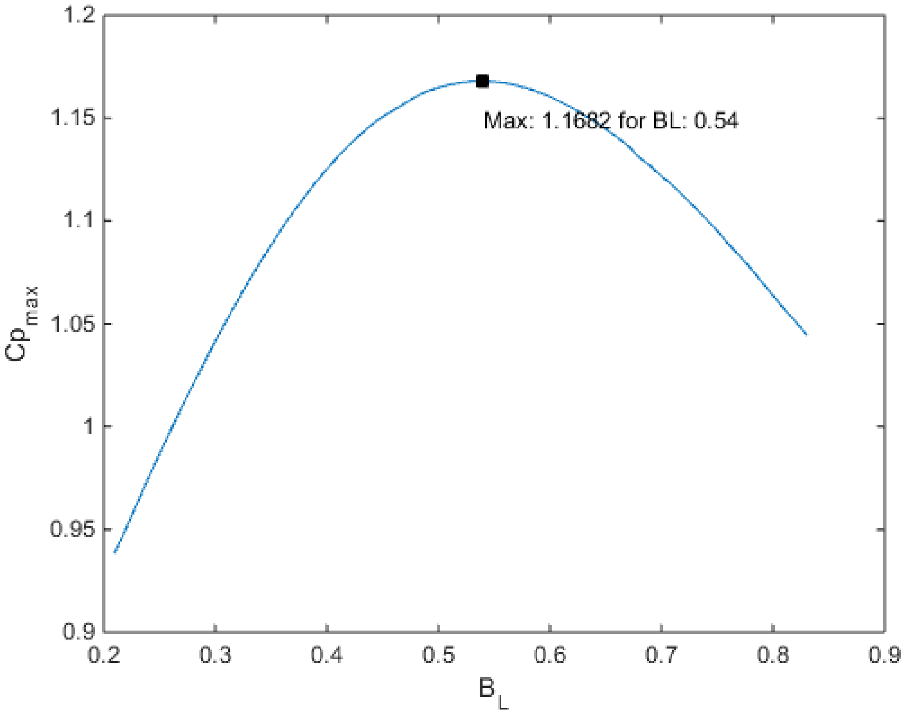

Figure 9 – C_{Pmax} versus B_L for B_G = 0.2