

1)

$$\frac{i-1}{i+1} = i$$

2)

$$1+i = \sqrt{2} e^{i45} \implies (1+i)^4 = 4 e^{i180} = -4$$

3)

$$(-3+2i)(2-i) = -4+7i = \sqrt{65} e^{i119.75}$$

4)

$$-4x^3 + 3x^2 + 25x + 6 = -(x-3)(4x^2 + 9x + 2)$$

Autres racines : $x = -2$ et $x = -\frac{1}{4}$

5)

$$\frac{3x^3 + 4x + 11}{x^2 - 3x + 2} = 3x + 9 + \frac{25x - 7}{x^2 - 3x + 2}$$

$$\frac{3x^3 + 4x + 11}{x^2 - 3x + 2} = 3x + 9 + \frac{A}{x-2} + \frac{B}{x-1} = 3x + 9 + \frac{43}{x-2} - \frac{18}{x-1}$$

6)

$$\lim_{x \rightarrow +\infty} \frac{(x-2)(2x^2+3)}{x(x-1)^2} = 2$$

$$\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{x^2 - 2x} = \lim_{x \rightarrow 2} \frac{(x+6)(x-2)}{x(x-2)} = \lim_{x \rightarrow 2} \frac{(x+6)}{x} = 4$$

$$\lim_{x \rightarrow -3} \frac{2x+1}{x+3} = -\infty$$

$$\lim_{x \rightarrow -\infty} \frac{8x^7 - 6x^3 + 4}{-9x^5 + 12x^3 - 6x} = -\infty$$

$$\lim_{x \rightarrow 0} \frac{\cos(2x) - 1}{x^3 + 5x^2} = \lim_{x \rightarrow 0} \frac{-2\sin(2x)}{3x^2 + 10x} = \lim_{x \rightarrow 0} \frac{-4\cos(2x)}{6x + 10} = -\frac{2}{5}$$

7)

$$\ddot{y}(t) - 3\dot{y}(t) + 2y(t) = e^{3t}$$

with: $y(0^-) = 1$; $\dot{y}(0^-) = 0$

$$(p^2 Y(p) - p) - 3(p Y(p) - 1) + 2Y(p) = \frac{1}{p-3}$$

$$(p^2 - 3p + 2) Y(p) = p - 3 + \frac{1}{p-3}$$

$$(p^2 - 3p + 2) Y(p) = \frac{p^2 - 6p + 10}{p-3}$$

$$Y(p) = \frac{p^2 - 6p + 10}{(p-3)(p^2 - 3p + 2)}$$

$$Y(p) = \frac{p^2 - 6p + 10}{(p-1)(p-2)(p-3)}$$

$$Y(p) = \frac{A}{p-1} + \frac{B}{p-2} + \frac{C}{p-3}$$

$$A = \frac{5}{2} \quad ; \quad B = -2 \quad ; \quad C = \frac{1}{2}$$

$$Y(p) = \frac{5}{2} \frac{1}{p-1} - 2 \frac{1}{p-2} + \frac{1}{2} \frac{1}{p-3}$$

$$y(t) = \frac{5}{2} e^t - 2 e^{2t} + \frac{1}{2} e^{3t}$$

8)

$$F(p) = \frac{2}{p^2 + 2p + 10} = \frac{2}{(p+1)^2 + 9} = \frac{2}{(p+1)^2 + 3^2} = \frac{2}{3} \frac{3}{(p+1)^2 + 3^2}$$

$$f(t) = \frac{2}{3} e^{-t} \sin(3t)$$