

1)

$$z_1 = i\sqrt{2}$$

$$|z_1| = \sqrt{2}$$

$$\theta_1 = \frac{\pi}{2}$$

$$z_1 = i\sqrt{2} = \sqrt{2}e^{90^\circ}$$

$$z_2 = 1 - i$$

$$|z_2| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\tan(\theta_2) = -1 \implies \theta_2 = \arctan(-1) = -\frac{\pi}{4}$$

$$z_2 = -1 + i = \sqrt{2}e^{-45^\circ}$$

$$\frac{z_1}{z_2} = e^{135^\circ}$$

2)

$$\frac{x^3 + 1}{x^2 - 1} = x + \frac{x + 1}{x^2 - 1} = x + \frac{x + 1}{(x + 1)(x - 1)} = x + \frac{1}{x - 1}$$

3)

$$\lim_{x \rightarrow -2} \frac{x^2 + x - 2}{2x^2 + 12x + 16} = \lim_{x \rightarrow -2} \frac{(x + 2)(x - 1)}{2(x + 2)(x + 4)} = -\frac{3}{4}$$

$$\lim_{x \rightarrow +\infty} \frac{1 - 3x}{5 - 4x} = \frac{3}{4}$$

4)

$$\frac{1}{x} > \frac{2}{5}$$

$$\frac{5 - 2x}{5x} > 0$$

$$\text{Tableau de signes} \implies S =]0; \frac{5}{2}[$$

$$\frac{2 - 6x}{3x - 2} \leq 0$$

$$\text{Tableau de signes} \implies S =]-\infty; \frac{1}{3}[\cup]\frac{2}{3}; +\infty[$$

5)

$$f_1(x) = e^{x^2+1}$$

$$f_1'(x) = 2xe^{x^2+1}$$

$$f_2(x) = \sqrt{\frac{2x}{3x+1}}$$

$$f_2'(x) = \frac{\frac{1}{(3x+1)^2}}{\sqrt{\frac{2x}{3x+1}}}$$

$$f_3(x, y) = x^2(x + y)$$

$$\frac{\partial f_3}{\partial x} = 3x^2 + 2xy$$

$$\frac{\partial f_3}{\partial y} = x^2$$

6)

$$\det(A - \lambda I) = 0 \implies \lambda^2 - 5\lambda + 4 = 0$$

2 valeurs propres distinctes : 1 et 4

$$A V = V \implies 2x = -y \implies V_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$A V = 4 V \implies x = y \implies V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix} \implies P^{-1} = \frac{1}{3} \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$$

$$D = P^{-1} A P = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$$

$$A = P D P^{-1} \implies A^{10} = P D^{10} P^{-1}$$

$$D^{10} = \begin{pmatrix} 1 & 0 \\ 0 & 1048576 \end{pmatrix} \implies A^{10} = \begin{pmatrix} 699051 & 349525 \\ 699050 & 349526 \end{pmatrix}$$