1)

$$z_1 = \frac{\sqrt{6} - i\sqrt{2}}{2}$$

$$|z_1| = \sqrt{\left(\frac{\sqrt{6}}{2}\right)^2 + \left(\frac{-\sqrt{2}}{2}\right)^2} = \sqrt{2}$$

$$\tan(\theta_1) = -\frac{\sqrt{2}}{\sqrt{6}} \Longrightarrow \theta_1 = \arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

$$z_1 = \frac{\sqrt{6} - i\sqrt{2}}{2} = \sqrt{2}e^{-30°}$$

$$z_2 = -1 + i$$

$$|z_2| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$\tan(\theta_2) = -1 \Longrightarrow \theta_1 = \arctan(-1) + \pi = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$$

$$z_2 = -1 + i = \sqrt{2}e^{135°}$$

$$\frac{z_1}{z_2} = e^{-165°} = e^{195°}$$

2)

$$\frac{2x^3 + 3x^2 - 44x - 97}{x^2 - 3x - 10} = 2x + 9 + \frac{3x - 7}{x^2 - 3x - 10} = 2x + 9 + \frac{8/7}{x - 5} + \frac{13/7}{x + 2}$$

3)

$$\lim_{x \to 4} \frac{x^2 - 8x + 16}{x^2 - 16} = \lim_{x \to 4} \frac{2x - 8}{2x} = 0$$

$$\lim_{x \to +\infty} x^2 - 3x = \lim_{x \to +\infty} x^2 \left(1 - \frac{3}{x}\right) = +\infty$$

4)

$$\frac{2x - 1}{x + 3} > \frac{2x}{x - 4}$$

$$\frac{2x - 1}{x + 3} - \frac{2x}{x - 4} > 0$$

$$\frac{-15x + 4}{(x + 3)(x - 4)} > 0$$

Tableau de signes $\Longrightarrow$ S=$]-\infty; -3[\cup \left]\frac{4}{15}; 4\right[$

5)

$$f_1(x) = \cos\left(\frac{2}{x}\right)$$

$$(\cos(u))' = -(\sin(u))u'$$

$$f_1'(x) = \frac{2}{x^2}\,\sin\left(\frac{2}{x}\right)$$

$$f_2(x, y) = x + e^y + 4y\cos(x) + x^3 + 6xy$$

$$\frac{\partial f_2}{\partial x} = 1 - 4y\sin(x) + 3x^2 + 6y$$

$$\frac{\partial f_2}{\partial y} = e^y + 4\cos(x) + 6x$$

6)

$$\int_0^1 \frac{3x+1}{(x+1)^2}dx = \int_0^1 \frac{3}{x+1} - \frac{2}{(x+1)^2}dx$$

$$\int \frac{1}{u} = [\ln|u|]$$

$$\int \frac{1}{u^2} = \left[-\frac{1}{u}\right]$$

$$\int_0^1 \frac{3x+1}{(x+1)^2}dx = 3\,[\ln|x+1|]_0^1 - 2\left[\frac{-1}{x+1}\right]_0^1 = 3\ln(2) - 1$$

$$\int_e^{e^3} \frac{dx}{x\ln(x)}$$

Posons $u = \ln(x) \Longrightarrow du = \dfrac{dx}{x}$

$$\int_e^{e^3} \frac{dx}{x\ln(x)} = \int_1^3 \frac{du}{u} = [\ln|u|]_1^3 = \ln(3)$$

7)

$$\det(B - \lambda I) = 0 \Longrightarrow \lambda^2 - \lambda - 2 = 0$$

2 valeurs propres distinctes : -1 et 2

$$B\,V = -V \Longrightarrow 2\,x = y \Longrightarrow V_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$B\,V = 2\,V \Longrightarrow x = y \Longrightarrow V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \Longrightarrow P^{-1} = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$D = P^{-1}\,B\,P = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$B = P\,D\,P^{-1} \Longrightarrow B^{10} = P\,D^{10}\,P^{-1}$$

$$D^{10} = \begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix} \Longrightarrow B^{10} = \begin{pmatrix} 2047 & -1023 \\ 2046 & -1022 \end{pmatrix}$$