1)

$$z_1\, z_2 = 13 + 11j$$

$$\frac{z_1}{z_2} = -\frac{17}{29} + \frac{1}{29}j$$

$$\frac{z_1^2}{3z_2} = -\frac{14}{87} + \frac{52}{87}j$$

	partie réelle	partie imaginaire	module	argument (en degrés)
z_1	1	-3	$\sqrt{10}$	-71,6°
z_2	-2	5	$\sqrt{29}$	111,8°
$z_1\, z_2$	13	11	$\sqrt{290}$	40,2°
$\frac{z_1}{z_2}$	$-\frac{17}{29}$	$\frac{1}{29}$	$\sqrt{\frac{10}{29}}$	176,6°
$\frac{z_1^2}{3z_2}$	$-\frac{14}{87}$	$\frac{52}{87}$	$\frac{10}{3\sqrt{29}}$	105,1°

2)

$$z_2 = 2\cos\left(-\frac{\pi}{6}\right) + 2\sin\left(-\frac{\pi}{6}\right) = \sqrt{3} - j$$

$$z_1 + z_2 = \left(-\sqrt{6} + \sqrt{3}\right) + \left(\sqrt{2} - 1\right) j = -0,7174 + 0,4142j$$

$$z_1 = 2\sqrt{2}\, e^{\frac{5\pi}{6} j} = 2\sqrt{2}\, e^{150^{\circ} j}$$

$$z_2 = 2\, e^{-\frac{\pi}{6} j} = 2\, e^{-30^{\circ} j}$$

$$z_1\, z_2 = 4\sqrt{2}\, e^{\frac{2\pi}{3} j} = 4\sqrt{2}\, e^{120^{\circ} j}$$

3) Division polynomiale

$$7x^5 + 4x^4 + 2x^3 - x + 5 = (7x^3 + 4x^2 - 12x - 8)\,(x^2 + 2) + 23x + 21$$

4)

$$\frac{x^3 - 2x + 4}{x^2 - 1} = x + \frac{-5/2}{x - 1} + \frac{3/2}{x + 1}$$

$$\frac{x+1}{(x-3)(x^2+x+2)} = \frac{4/14}{x-3} + \frac{-4/14x-11/21}{x^2+x+2}$$

5)

$$\lim_{x\to1} \frac{-2x^2-x+3}{x-1} = \lim_{x\to1} \frac{-2(x-1)(x+\frac{3}{2})}{x-1} = -5$$

$$\lim_{x\to+\infty} \frac{4x(-x-1)}{(x^2+2)(x+3)} = 0^-$$

$$\lim_{x\to+\infty} \frac{\sqrt{x}+2-3x}{x} = \lim_{x\to+\infty} \frac{1}{\sqrt{x}} + \frac{2}{x} - 3 = -3$$

6)

$$\frac{1}{x} + \frac{2}{x+1} = 0$$

S=$\left\{-\frac{1}{3}\right\}$

$$\frac{4(x^2-25)}{2x+10} = 0$$

S=$\{5\}$

$$\frac{3x+1}{2-x} \geq 0$$

Tableau de variation $\Longrightarrow$ S=$\left[-\frac{1}{3};2\right[$

$$(3x-12)^2 \leq (-2x+7)^2$$

$$(3x-12)^2 - (-2x+7)^2 \leq 0$$

$$(3x-12-2x+7)(3x-12+2x-7)^2 \leq 0$$

$$(x-5)(5x-19) \leq 0$$

Tableau de variation $\Longrightarrow$ S=$\left[\frac{19}{5};5\right]$

7)

$$f'(x) = 36x^2 - 8x + 6$$

$$g'(x) = \frac{3x+2}{2x\sqrt{x}}$$

8)

$$\begin{aligned}
\int_1^2 \frac{x^2+3x+1}{x^2}dx &= \int_1^2 \left(1+\frac{3}{x}+\frac{1}{x^2}\right)dx \\
&= \left[x+3\ln(x)-\frac{1}{x}\right]_1^2 \\
&= 2+3\ln(2)-\frac{1}{2}-1-3\ln(1)+1 \\
&= \frac{3}{2}+3\ln(2)
\end{aligned}$$

9)

$$\ddot{y}(t)+y(t)=\sin(2t)$$

$$\ddot{y}(t) \longrightarrow p\left(pY(p)-y(0^-)\right)-\dot{y}(0^-)=p(pY(p)-2)-1$$

$$p^2Y(p)-2p-1+Y(p)=\frac{2}{p^2+4}$$

$$(p^2+1)Y(p)=\frac{2}{p^2+4}+2p+1$$

$$Y(p)=\frac{2}{(p^2+4)(p^2+1)}+\frac{2p}{p^2+1}+\frac{1}{p^2+1}$$

$$\frac{2}{(p^2+4)(p^2+1)}=\frac{Ap+B}{p^2+4}+\frac{Cp+D}{p^2+1}$$

$$A=0 \qquad ; \qquad C=0 \qquad ; \qquad B=-D=-\frac{2}{3}$$

$$\frac{2}{(p^2+4)(p^2+1)}=\frac{-2/3}{p^2+4}+\frac{2/3}{p^2+1}$$

$$Y(p)=\frac{-2/3}{p^2+4}+\frac{2/3}{p^2+1}+\frac{2p}{p^2+1}+\frac{1}{p^2+1}$$

$$Y(p)=-\frac{1}{3}\frac{2}{p^2+4}+\frac{2/3}{p^2+1}+\frac{2p}{p^2+1}+\frac{1}{p^2+1}$$

$$y(t)=-\frac{1}{3}\sin(2t)+\frac{2}{3}\sin(t)+2\cos(t)+\sin(t)$$

$$y(t)=-\frac{1}{3}\sin(2t)+\frac{5}{3}\sin(t)+2\cos(t)$$