

1)

$$\frac{2}{1+i} - \frac{3}{1-i} = \frac{2(1-i) - 3(1+i)}{(1+i)(1-i)} = \frac{-1-5i}{2} = -\frac{1}{2} - \frac{5}{2}i$$

2)

$$z = 2 + 2i = 2\sqrt{2}e^{i\pi/4}$$

$$z^6 = 2^9 e^{i3\pi/2} = -512i$$

3)

$$x^3 + 6x^2 + 5x - 12 = (x-1)(ax^2 + bx + c)$$

Division polynomiale :

$$x^3 + 6x^2 + 5x - 12 = (x-1)(x^2 + 7x + 12)$$

On résoud le polynôme d'ordre 2 et on trouve 2 racines réelles : -3 et -4

$$x^3 + 6x^2 + 5x - 12 = (x-1)(x+3)(x+4)$$

4)

$$(x-1-i)(x-1+i)(x+2) = x^3 - 2x + 4$$

5)

$$\frac{2x^3 + x^2 - x - 1}{x^2 - 2x + 1} = 2x + 5 + \frac{7x - 4}{(x-1)^2}$$

$$\frac{2x^3 + x^2 - x - 1}{x^2 - 2x + 1} = 2x + 5 + \frac{3}{(x-1)^2} + \frac{7}{x-1}$$

6)

$$\lim_{x \rightarrow -\infty} \frac{x^3 - 2x + 10}{5x^3 + 3x - 2} = \frac{1}{5}$$

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} x + 2 = 4$$

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x} = \lim_{x \rightarrow 0} \frac{-\sin(x)}{1} = 0$$

7)

$$(3x - 5)(6 + 3x) = 0$$

$$S = \left\{ -2; \frac{5}{3} \right\}$$

$$\frac{5x-1}{x-3}=0$$

$$S=\left\{\frac{1}{5}\right\}$$

$$2x^2+5=1$$

$$2x^2=-4$$

$$S=\emptyset$$

$$(6-2x)(x+1)\geq 0$$

$$\text{Tableau de variation} \implies S=[-1;3]$$

$$\frac{2-x}{2x-8}\leq 0$$

$$\text{Tableau de variation} \implies S=]-\infty;2]\cup]4;+\infty[$$

8)

$$f(x)=\sqrt{x^2-3x+2}$$

$$f'(x)=\frac{2x-3}{2\sqrt{x^2-3x+2}}$$

$$f(x)=\sin(x^2)$$

$$f'(x)=2x\cos(x^2)$$

9)

$$\int_0^\pi \sin(x)dx=[-\cos(x)]_0^\pi=-\cos(\pi)+\cos(0)=2$$

10)

$$\dot{y}(t)+y(t)=1$$

$$\text{avec : } \quad y(0^-)=-1$$

$$pY(p)+1+Y(p)=\frac{1}{p}$$

$$(p+1)Y(p)=\frac{1}{p}-1$$

$$Y(p)=\frac{1}{p(p+1)}-\frac{1}{p+1}=\frac{1}{p}-\frac{1}{p+1}-\frac{1}{p+1}=\frac{1}{p}-\frac{2}{p+1}$$

$$y(t)=1-2e^{-t}$$

11)

$$Y(p) = \frac{p}{(p+1)(p^2+1)} = \frac{1}{2} \left(\frac{p+1}{p^2+1} - \frac{1}{p+1} \right)$$

$$y(t) = \frac{1}{2} \left(\cos(t) + \sin(t) - e^{-t} \right)$$