1)

$$\frac{1+i}{2-i} = \frac{(1+i)(2+i)}{(2-i)(2+i)} = \frac{1+3i}{5}$$

En élevant au carré :

$$\left(\frac{1+i}{2-i}\right)^2 = \frac{-8+6i}{25} = \frac{-8}{25} + \frac{6}{25}i$$

2)

$$-i\sqrt{2} = \sqrt{2}\, e^{i\frac{3\pi}{2}}$$

$$i+1 = \sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$z = \frac{-i\sqrt{2}}{1+i} = e^{i(\frac{3\pi}{2}-\frac{\pi}{4})} = e^{i\frac{5\pi}{4}}$$

On en déduit que :

$$z^2 = \left(e^{i\frac{5\pi}{4}}\right)^2 = e^{i\frac{5\pi}{2}} = i$$

3)

$$2x^3 - 10x^2 + 14x + 26 = 2(x^3 - 5x^2 + 7x + 13) = 2(x+1)(ax^2 + bx + c)$$

Division polynomiale :

$$x^3 - 5x^2 + 7x + 13 = (x+1)(x^2 - 6x + 13)$$

On résoud le polynôme d'ordre 2 et on trouve 2 racines complexes conjuguées : $(3 \pm 2i)$

$$2x^3 - 10x^2 + 14x + 26 = 2(x+1)(x-3+2i)(x-3-2i)$$

4)

$$(x+3)(x-2)(x-1-i)(x-1+i) = x^4 - x^3 - 6x^2 + 14x - 12$$

5)

$$\frac{x^2-2x-37}{x^2-3x-40} = 1 + \frac{x+3}{(x+5)(x-8)} = 1 + \frac{2/13}{x+5} + \frac{11/13}{x-8}$$

6)

$$\lim_{x\to 5} \frac{3x+2}{6x-4} = \frac{17}{26}$$

$$\lim_{x\to 7} \frac{1}{x-7} = \pm\infty$$

$$\lim_{x\to +\infty} \frac{2x^2+5x+1}{6x^2-5} = \frac{1}{3}$$

$$\lim_{x\to 5} \frac{\sqrt{x-1}-2}{x-5} = \lim_{x\to 5} \frac{1}{2\sqrt{x-1}} = \frac{1}{4}$$

7)

$$x^4 + x^2 - 12 = 0$$

$$X^2 + X - 12 = 0$$

2 solutions : $X_1 = 3$ et $X_2 = -4$
2 solutions dans $\mathbb{R}$ : $\pm\sqrt{3}$

$$(x+4)^2 = 25$$

$$(x+4)^2 - 25 = 0$$

$$(x+4+5)(x+4-5) = 0$$

$$(x+9)(x-1) = 0$$

S=$\{-9; 1\}$

$$(2x+1)(1-3x) \geq 0$$

Tableau de variation $\Longrightarrow$ S=$\left[-\frac{1}{2}; \frac{1}{3}\right]$

8)

$$f(x) = (2x^2+3x)^4$$

$$f'(x) = 4(4x+3)(2x^2+3x)^3$$

$$f(x) = \cos\left(\frac{1}{x}\right)$$

$$f'(x) = \frac{1}{x^2}\,\sin\left(\frac{1}{x}\right)$$

9)

$$\int_1^4 \frac{3}{x^2}dx = \left[-\frac{3}{x}\right]_1^4 = \frac{9}{4}$$

10)

$$\dot{y}(t) = y(t) + t\,e^t$$

$$p\,Y(p) + 1 = Y(p) + \frac{1}{(p-1)^2}$$

$$Y(p) = \frac{1}{(p-1)^3} - \frac{1}{p-1}$$

$$y(t) = \frac{t^2}{2}\,e^t - e^t = \left(\frac{t^2}{2} - 1\right)\,e^t$$