1)

$$det(A - \lambda I) = \begin{vmatrix} -1-\lambda & 0 & 1 \\ 2 & -\lambda & 2 \\ 2 & 0 & -\lambda \end{vmatrix} = (-1-\lambda)\lambda^2 + 2\lambda = \lambda(-\lambda^2 - \lambda + 2)$$

3 valeurs propres simples : -2, 0, 1

La matrice a 3 valeurs propres distinctes, donc elle est diagonalisable.

$$A\,u = -2u \Longrightarrow -x + z = -2x; 2x + 2z = -2y; 2x = -2z \Longrightarrow z = -x; y = 0$$

$$u = \begin{pmatrix} x \\ 0 \\ -x \end{pmatrix} = x \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$A\,u = 0 \Longrightarrow -x + z = 0; 2x + 2z = 0; 2x = 0 \Longrightarrow x = 0; z = 0$$

$$u = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} = y \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$A\,u = u \Longrightarrow -x + z = x; 2x + 2z = y; 2x = z \Longrightarrow z = 2x; y = 6x$$

$$u = \begin{pmatrix} x \\ 6x \\ 2x \end{pmatrix} = x \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 6 \\ -1 & 0 & 2 \end{pmatrix}$$

2)

$$D = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

On peut calculer P^{-1} et vérifier que $D = P^{-1}AP$.

$$P^{-1} = \frac{1}{3} \begin{pmatrix} 2 & 0 & -1 \\ -6 & 3 & -6 \\ 1 & 0 & 1 \end{pmatrix}$$