1)

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix}$$

$$a = 0 \qquad ; \qquad d = 0 \qquad ; \qquad b = -c$$

Exemple de matrice antisymétrique :

$$B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

2)

$$\det(A - \lambda I) = 0 \Longrightarrow \lambda^2 - 4\lambda - 5 = 0$$

2 valeurs propres distinctes : -1 et 5

$$A\,V = -V \Longrightarrow x = -2y \Longrightarrow V_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$A\,V = 5\,V \Longrightarrow x = y \Longrightarrow V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \Longrightarrow P^{-1} = \frac{1}{3}\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$$

$$D = P^{-1}\,A\,P = \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$$

3)

$$A = P\,D\,P^{-1} \Longrightarrow A^4 = P\,D^4\,P^{-1}$$

$$D^4 = \begin{pmatrix} 1 & 0 \\ 0 & 625 \end{pmatrix} \Longrightarrow A^4 = \begin{pmatrix} 209 & 416 \\ 208 & 417 \end{pmatrix}$$