

1) la matrice A est diagonale

$$A^{10} = \begin{pmatrix} (-1)^{10} & 0 \\ 0 & 2^{10} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix}$$

2) il faut diagonaliser la matrice B .

$$\det(B - \lambda I) = 0 \implies \lambda^2 - 5\lambda + 4 = 0$$

2 valeurs propres distinctes : 1 et 4

$$B V = V \implies 2x = -y \implies V_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$B V = 4 V \implies x = y \implies V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix} \implies P^{-1} = \frac{1}{3} \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$$

$$D = P^{-1} B P = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$$

$$B = P D P^{-1} \implies B^{10} = P D^{10} P^{-1}$$

$$D^{10} = \begin{pmatrix} 1 & 0 \\ 0 & 1048576 \end{pmatrix} \implies B^{10} = \begin{pmatrix} 699051 & 349525 \\ 699050 & 349526 \end{pmatrix}$$