1)

$$\det(A) = -1(5-4) + 2(10-3) + 2(8-3) = 23$$

2) On calcule d'abord la matrice des cofacteurs :

$$\begin{pmatrix} \begin{vmatrix} 1 & 1 \\ 4 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} \\ \begin{vmatrix} -2 & 2 \\ 4 & 5 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix} \\ \begin{vmatrix} -2 & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 7 & 5 \\ -18 & -11 & 2 \\ -4 & -5 & 3 \end{pmatrix} \times \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} = \begin{pmatrix} 1 & -7 & 5 \\ 18 & -11 & -2 \\ -4 & 5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{23} \begin{pmatrix} 1 & 18 & -4 \\ -7 & -11 & 5 \\ 5 & -2 & 3 \end{pmatrix}$$

3)

$$\det(B - \lambda I) = 0 \Longrightarrow \lambda^2 - \lambda - 2 = 0$$

2 valeurs propres distinctes : -1 et 2

$$B\,V = -V \Longrightarrow 2\,x = y \Longrightarrow V_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$B\,V = 2\,V \Longrightarrow x = y \Longrightarrow V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \Longrightarrow P^{-1} = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$D = P^{-1}\,B\,P = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$$

4)

$$B = P\,D\,P^{-1} \Longrightarrow B^{10} = P\,D^{10}\,P^{-1}$$

$$D^{10} = \begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix} \Longrightarrow B^{10} = \begin{pmatrix} 2047 & -1023 \\ 2046 & -1022 \end{pmatrix}$$